vector quantities have the further property of standing perpendicular to a system of surfaces, viz, the system of level surfaces, or isobaric surfaces, respectively. But the magnitudes of these quantities can no longer be expressed by the thickness of the lamella between the successive surfaces, but for this purpose we must introduce a second system of surfaces, viz, the surfaces of equal density, or equal specific volume. These two vector quantities therefore belong to the category of complex lamellar vector quantities that Lord Kelvin has so called on account of the two systems of surfaces and lamellæ. Therefore, in the strict sense of the word, these quantities are not of the nature of gradients. The importance of these vector quantities depends especially upon the fact that they are the simplest quantities upon which we can base the computation of the circulatory motions of the atmosphere. The accelerating barometric gradient is the vector quantity whose tangential component appears under the sign of integration in equation (8), and the tangential component of the weight of a unit volume of air will occur under the integration sign in the corresponding integral when we describe the circulation by the use of the specific moving quantities, viz, the product of velocity and density, instead of by velocity alone.

By the combination of a simple lamellar vector quantity (the barometric gradient) and a complex lamellar vector (the weight of the air in a unit of volume) there arises that vector quantity which Möller has called "the space gradient," *Gr*.

It therefore seems to me important to earnestly warn against the use of the term *gradient* for this vector quantity. Since, misled by the name gradient, we are liable to attribute to it the properties of a gradient, and be led into further error with respect to the isostenic surfaces. Moreover, a special name for this quantity is entirely unnecessary, for in mathematical relations it is nothing else than a vector quantity, and in mechanical aspects it is a force of a most general nature. The retention of the name gradient would also be equivalent to saying that in meteorology we call that a gradient which in mechanics is called force, and that in meteorology we speak of magnitudes of the nature of the gradient, when the mathematician speaks of vector quantities. This special name can be useful only when we apply the term gradient in a mechanical sense to forces of a very special nature, and in a mathematical sense to vectors of a very special nature, and then alone would the name have a prospect of being accepted from meteorology into the mathematical and mechanical sciences, and of assisting instead of hindering the cooperation of these sciences in meteorological questions.

The rational introduction of a terminology will, therefore, encounter no serious difficulty where the term gradient is used only in the above-mentioned special sense; but so far as I know no other meteorologist has accepted and used this special gradient. Only in one point do we come into disagreement with the old usage, viz, by the term vertical gradient we generally mean, not the vertical gradient of pressure, but the difference between the vertical gradient of pressure and the force of gravity. This is the vertical component of Möller's gradient. So long as we retain this term, instead of speaking of the vertical force, it will be very easy to call the general force directed at random in space, the *space* gradient, and misled by this name to attribute to this force the properties of the gradients and thereby again find ourselves tending toward the errors with regard to isostenic surfaces. Therefore, it would seem best not to make any further use of the term vertical gradient in the above sense, which is in fact not generally done, but to indicate the difference between the two forces by the simple term "vertical force."

LINE INTEGRALS IN THE ATMOSPHERE.

By FRANK H. BIGELOW.

The papers by Prof. V. Bjerknes[1], in connection with a criticism thereon[2] by Dr. M. Möller, and a practical application of the theory to a cyclone, by Dr. J. W. Sandström[3], have brought before meteorologists a problem of interest and practical importance. It is, therefore, desirable to investigate the bearings of the theory from various points of view. The following contribution to the subject is not intended as a criticism of the preceding works, but as a supplement to the discussion of the subject given by Prof. V. Bjerknes himself.[4]

What the theory is may readily be described in the following manner: The well known diagram by Hertz of the adiabatic changes in the condition of moist air, Abbe's translations, On the Mechanics of the Earth's Atmosphere, shows the relations between pressure B, temperature t, and vapor tension e, in the four stages, α, β, γ, δ. Now, since the density ρ is a function of B, t, e, only, a similar diagram will result for the function ρ, the density, and hence for the specific volume $v = \frac{1}{\rho}$, by drawing other lines on the same coordinate axes.

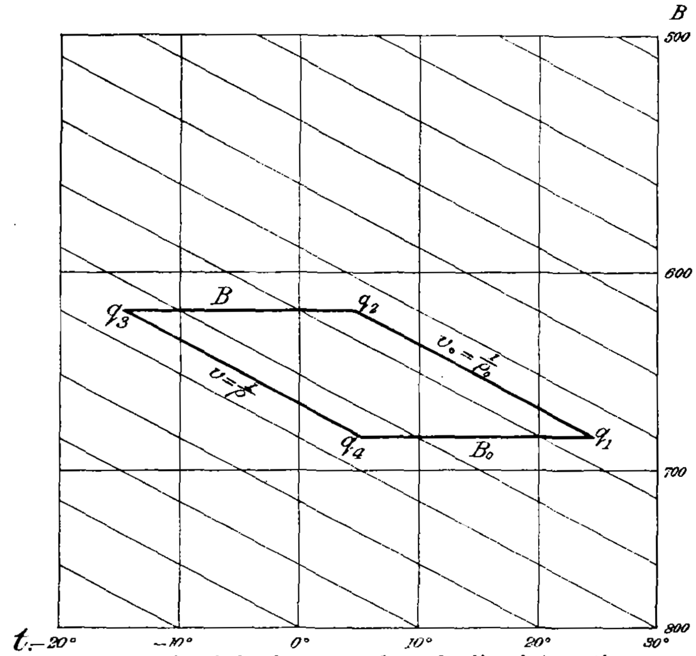

FIG. 1.—Circuit in the atmosphere for line integration.

Using throughout this paper the notation laid down in my International Cloud Observations Report, on pages 485–488, and also the formulæ derived in the sections following, we have

$$\frac{1}{\rho} = \frac{P_o(1+\alpha t)}{\rho_o P} \text{ for dry air,} \qquad 47a;$$

$$\text{and } \frac{1}{\rho} = \frac{1}{\rho_o}\frac{B_o}{B}(1+\alpha t)(1+\beta)(1+\gamma)\left(1+\frac{h+h_o}{R}\right).$$

$$= \frac{1}{\rho_o}\frac{760}{(B-.377e)}\cdot\frac{1+\alpha t}{n_1}, \text{ for moist air.}$$

[1] Das dynamische Princip. der Circulations bewegungen in der Atmosphäre. Meteorol Zeit. Marz 1900 und April 1900. Translation. WEATHER REVIEW, October, 1900.

[2] Der räumliche Gradient, M. Möller. Meteorol Zeit. June, 1900.

[3] Ueber die Verwendung von Prof. V. Bjerknes' Theorie der Wirbelbewegungen in Gasen und Flüssigkeiten. Königl. Schwed. Ak. der Wiss. January, 1900.

[4] Räumlicher Gradient und Circulation. Von V. Bjerknes. Meteorol. Zeits. November, 1900.

It will be observed that Sandström seems to have omitted the following small variations depending upon gravity,

$$\frac{1}{n_1} = (1 + \gamma)\left(1 + \frac{h + h_o}{R}\right).$$

If we make the following changes in notation Sandström's formulæ become the same as those of the Standard system:

Sandström.		Bigelow.
Pressure,	p.	P.
Temperature,	t.	t.
Relative humidity,	r.	$R.\ H.$
Vapor tension,	f.	e (t and $g = 980.6$).
Coef. of expansion,	α.	$\alpha = .00367$.
Normal density,	c.	$\rho_o\ (t = 0, B_n = 760)$.
Other density,	q.	ρ (B, t, $R.\ H.$, as assigned).

By computing and plotting the lines of equal volume $v = \frac{1}{\rho}$ on the coordinates of temperature and logarithm of pressure, we have such diagonals as appear on the scheme of fig. 1 which is not drawn to scale. Taking such a circuit as is indicated by $B_o\ v_o\ B\ v$, Bjerknes has shown that the line integral of the circulation is $A = \frac{dC}{dt} = (P - P_o)\ (v - v_o)$, which is the work done in moving the unit mass once around the circuit. For, a certain amount of work is expended against gravity in raising the unit mass from q_1 to q_2 along the line of equal density ρ_o, no work is done in moving it along the isobaric surface B from q_2 to q_3, but work is gained in the fall of the mass from B to B_o along the line ρ from q_3 to q_4, and none in moving it along the surface B_o from q_4 to q_1. The sum of the work along the lines $\frac{1}{\rho_o}$ and $\frac{1}{\rho}$ taken algebraically, if not equal to zero is equivalent to $4\pi I$, a well-known theorem in dynamics, since it is the curl of the total current through the circuit. The only objection to this procedure is the practical difficulty involved in computing the values of ρ_o and ρ in the free atmosphere. For this purpose it is proposed to utilize actual measures as provided by kite and balloon ascensions, where the observations are to be taken *in situ*, and then combined through computations as illustrated by Sandström.

The purpose of this paper is to show that the same result can be obtained in a different way, and that this involves only such observations on the velocities of the motions of the air as may be secured by the ordinary triangulations with two theodolites placed at the extremities of a base line, as was done in making the international cloud observations in 1896–1897.

The definition of work is that it is the product of the force of acceleration and the distance through which it acts. Thus, if the gradient pressure force is $-\frac{1}{\rho}\frac{\partial P}{\partial x}$, and the distance ∂x, the work $= -\frac{1}{\rho}\frac{\partial P}{\partial x} . \partial x = -\frac{\partial P}{\rho}$. If this is computed along three rectangular axes joining the points $x_o\ y_o\ z_o$ and $x\ y\ z$, the sum is the work required to move a unit mass from the first to the second point. Now, the work, as derived from the equations of motion, involving acceleration of velocity, centrifugal forces, and deflecting forces on the rotating earth, is given in three coordinates by formula 210 as follows:

$$-\frac{\partial P}{\rho} = \frac{dx}{dt} . \partial u - 2n \cos\theta\, v . \partial x - \frac{\cot\theta}{r} v^2 . \partial x + \frac{1}{r} u . w . \partial x.$$

$$-\frac{\partial P}{\rho} = \frac{dy}{dt} . \partial v + 2n \sin\theta\, w . \partial y + \frac{1}{r} v . w . \partial y + 2n\cos\theta\, u . \partial y + \frac{\cot\theta}{r} u\, v . \partial y.$$

$$-\frac{\partial P}{\rho} = \frac{dz}{dt} . \partial w - 2n \sin\theta\, v . \partial z - \frac{1}{r} v^2 . \partial z - \frac{1}{r} u^2 \partial z + g \partial z.$$

This is derived from 155 by making the substitutions indicated in the following pages up to 505. If we introduce

202, $\qquad v\,\partial x = u\,\partial y, \quad w\,\partial x = u\,\partial z, \quad w\,\partial y = v\,\partial z,$

and take the sum of the partial differentials, the result is

203, $\qquad -\frac{\partial P}{\rho} = u\,\partial u + v\,\partial v + w\,\partial w + g\,\partial z.$

We have for moist air mixed with dry air, 47*a*,

$$\frac{1}{\rho} = \frac{1}{\rho_o}\frac{P_o\,(1 + \alpha t)\,(1 + \beta)}{P} = \frac{1}{aP}.$$

For common logarithms,

$$\frac{1}{a} = \frac{P_o\,(1 + \alpha t)\,(1 + \beta)}{\rho_o M}.$$

$$= g_o\frac{\rho_m\ B_n\ (1 + \alpha t)\,(1 + \beta)}{\rho_o M} = \frac{(1 + \alpha t)\,(1 + \beta)}{a_o}$$

$$\frac{1}{a_o} = \frac{78359}{M} = 180431 = Kg_o = 18400 \times 9.806.$$

$$= g_o\frac{l}{M} = \frac{l}{M} g\,(1 + \gamma)\,(1 + \delta). \quad \text{Hence,}$$

$$\frac{1}{a} = gK\,(1 + \alpha t)\,(1 + \beta)\,(1 + \gamma)\,(1 + \delta).$$

Furthermore, we have

$$\log\frac{P_o}{P} = (1 + \gamma)\frac{B_o}{B}. \qquad 57$$

Compare the general formula 58. Now, integrating 203 along a given line, where ρ is constant, we obtain,

$$-\int\frac{\partial P}{\rho} = -\frac{1}{a}\int\frac{\partial P}{P} = -\frac{\log P}{a}$$

$$= \frac{u^2}{2} + \frac{v^2}{2} + \frac{w^2}{2} + gz = \frac{q^2}{2} + gz.$$

Placing the limits at the isobars P and P_o there results along ρ_o, by distinguishing the points as q_1 and q_2,

$$-\int\frac{dP}{\rho_o} = -\frac{(P - P_o)}{\rho_o} = \frac{\log P_o - \log P}{a}$$

$$= \tfrac{1}{2}\,(q_2^2 - q_1^2) + g\,(z_2 - z_1).$$

This is the work done in moving the unit mass from q_1 to q_2 If we integrate similarly along the line ρ, we have

$$-\frac{(P - P_o)}{\rho} = \tfrac{1}{2}\,(q_3^2 - q_4^2) + g\,(z_3 - z_4).$$

If the integral sum is taken algebraically around the circuit in one direction, since the terms along the isobars having $(P - P) = 0$ and $(P_o - P_o) = 0$ of course disappear, there remains,

$$(P - P_o)\left(\frac{1}{\rho} - \frac{1}{\rho_o}\right) = (P - P_o)\ (v - v_o)$$

$$= (\log P_o - \log P)\ \left(\frac{1}{a_1} - \frac{1}{a_2}\right)$$

$$= \tfrac{1}{2}\,(q_3^2 + q_2^2 - q_4^2 - q_1^2) + g\,(z_3 + z_2 - z_4 - z_1).$$

Hence, we see that the Bjerknes theorem is reproduced, and that it may be expressed as the product of the difference of two pressures multiplied by the difference of the two volumes; or as the product of the difference of two logarithms by the difference of the reciprocal of two coefficients; or as one-half the sum of the squares of the velocities at two points on the upper isobar, minus the sum of the squares of the velocities at two points on the lower isobar, added to the product of gravity by the sum of the heights of the same two points on the upper isobar, minus the heights of the two points on the lower isobar. The pressures P, P_0 can be changed into the barometric pressures B, B_0 by adding the factor $(1 + \eta)$.

We see then that instead of computing the work integral around the circuit by means of the densities as measured by observations *in situ*, the velocities measured by theodolites stationed on the ground are a perfect substitute, and the mode of obtaining the velocities of the motion of the air at four points by triangulation is much easier than by ascensions of any kind. Furthermore, it is not necessary that the upper and the lower point-pairs should lie on the same isobar, for it is equally correct to use any four points, and indeed any number of points joined together in a chain around a circuit in the atmosphere, by combining the velocities and the gravity terms in algebraic succession. Attention is called to the fact that the recent Report of the Chief of the Weather Bureau on the International Cloud Observations contains a large amount of material in proper form for such computations. The twenty subareas of the normal cyclones and anticyclones give at the heights of the designated cloud strata the velocities of motion carefully determined. They are also arranged in vertical lines over each subarea, so that there are 20 vertical gagings through each cyclone and anticyclone, respectively. Also, the data for computing the specific volumes are given in the same connection, and can be employed in check computations. We hope to be able to pay some further attention to the development of this interesting subject.

It is noted that in the course of the integration all the terms depending upon deflecting and centrifugal forces have dropped out. Bjerknes properly stated that these could be neglected. In the final equation, if the integration is around a circuit, the terms in gz will disappear because the sum of the (z) is zero, the involved variations of the products (gz) being too small to have appreciable value. Finally, by 389a, page 588, it is seen that the temperature gradients involve the form $(z - z_0)$ as a factor, so that in a circuit this correction practically disappears. There remains only the friction kq, and as this is an insignificant quantity in the air above a thin surface skin, say 500 feet deep, but little will be derived from this term to modify the circulation. On the other hand, turbulent motions and the interplay of mixing minor vortices in all stages of formation may actually introduce a term of significance. These questions, and many more, can be discussed to advantage by the theory of closed circuits. When the integration is not around a circuit the gravity temperature and friction terms must enter the equations. It is also to be remarked that reductions to sea level, or from one height to another in the free air, are not complete when referred to the static term gz alone, as is commonly done in barometry, but the dynamic term $\frac{1}{2}\, q^2$ should also enter. The United States Weather Bureau is engaged in reconstructing its entire series of barometric observations, taken during the past thirty years, and they are to be reduced to a homogeneous system. It can now be stated that there still remain some small residuals, which may be due to local variations of gravity, or to local variations of the assumed mean temperature of the air column, or to local changes in a constant used for the plateau reductions, or finally they may be due in part to the swirl of local circulations or eddies, all of which the theory under discussion may be used in elucidating. It is an important advance in meteorology to have the problem, as stated by Prof. V. Bjerknes and Dr. Sandström, brought to the attention of students of the dynamics of the atmosphere.

THE PEOPLE OF MARS.

By CHARLES FITZHUGH TALMAN, U. S. Weather Bureau.

The climates of other planets than our own form a subject which will perhaps largely occupy the attention of future meteorologists who, with more efficient means of observation than we now possess, may find in the phenomena of the planetary atmospheres important aid in the elucidation of many obscure phases of their science.

Already, in the study of the planet Mars, certain interesting and seemingly anomalous atmospheric conditions have been observed. The insolation of this planet, per unit of area, is less than half that of the earth. This circumstance would seem to imply a rigorous climate, yet it is almost certain that the average temperature at the Martian surface is somewhat higher than that of our own globe; for otherwise we can hardly account for the small extent of the polar snow-caps which are so important a feature of the planet's topography.

All the evidence points to a similarity between the terrestrial and Martian climates, and this fact leads up to the perennial topic of the planet's inhabitants. Never does there occur a favorable opposition of Mars, that the newspapers do not seize the opportunity to publish more or less fantastic dissertations on this inexhaustible subject, in which the sometimes relatively sober and moderate intention of the writers is completely misinterpreted by extravagant pictures, purporting to be faithful portrayals of the Martians and of the conditions of life on their planet. At the same time hare-brained speculators spring up everywhere prepared to show us exactly how to telegraph across the abyss of space and communicate with the inhabitants of our sister world.

It is strange that no one has ever pointed out how unphilosophical, from a biological point of view, is the question, "Are there people on Mars?"

In the writings of many latter-day litterateurs and not a few professed scientists we find glib references to the people of this or that planet, and the use of these words implies the assumption that life, from its necessarily simple beginnings, has, in each world where it exists, ultimately developed one and only one species more or less like the human race, and clearly differentiated from and superior to all the other species produced in the same world. But nothing that we know of the evolutionary process warrants such an assumption.

The imaginable forms which living matter may assume are infinitely diverse. Look forth upon the myriad species of organic beings—plants and animals—which our own world contains. Where among them all can you find an organism, other than man, which, if placed upon Mars, would be intellectually capable of communicating with us? What success would we have in attempting to telegraph to a race of horses or guinea pigs, for example?

On our own planet the development of life apparently entered, at an early stage, upon two diverse roads. The forms subsequently evolved, though probably of common ancestry, are nevertheless clearly and naturally divided into two great kingdoms, the animal and the vegetal. But there is no reason for supposing that the course of events has been the same in other worlds than ours. For example, it may be that on Mars plant life only exists. Now, suppose that, as the speculators on this subject commonly assume, Mars has supported life longer than the earth. In such a case the plant forms would presumably have reached a high stage of development; plants would there exist compared with which our highest plants, such as daisies and asters, are simple and rudimentary. Nevertheless, it is not conceivable that any plant, however high in the scale, could hold communication with the human race.

However, I think it is most reasonable to suppose that, if